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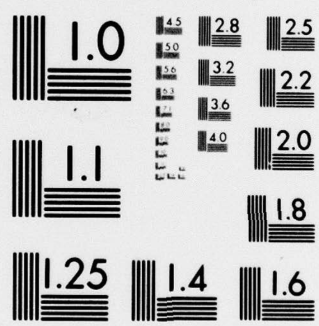

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A NOTE ON INITIAL FILL RATE

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A common measure of customer service is initial fill rate, the percent of demand satisfied without delay from stock on hand. An exact expression for initial fill rate when demand is Compound Poisson has previously been developed. This note presents an approximation which is simple to program and relatively quick to run. Its accuracy is distribution dependent. For Negative Binomial distribution of demand, its accuracy is evaluated and found to be good. A common alternative approach to calculate initial fill is shown to work very badly for one class of items.

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A NOTE ON INITIAL FILL RATE

1. Summary

A common measure of customer service is initial fill rate, the percent of demand satisfied without delay from stock on hand. Silver has developed an exact expression for initial fill rate when demand is Compound Poisson. This note presents an approximation which is far simpler to program, and much less time consuming to run. Its accuracy is distribution dependent. For Negative Binomial distribution of demand, its accuracy is evaluated and found to be good. The Negative Binomial was chosen because of its use in a standard US Army inventory system (3), and its general suitability for modelling demand.

Other assumptions are that unfilled demand is backlogged, that the Negative Binomial parameters are known, and that a continuous review reorder point, order-up-to-level policy is used. Because transaction size may exceed one, inventory at the time of an order may be below the reorder point. The difference is termed the deficit.

Notation:

R - reorder point

M - order-up-to-level

Q - reorder quantity for zero deficit (M-R)

Z - deficit

X - lead time demand

S - transaction size

f(x), t(z), (h(s) are probability mass functions for X, Z, S.

IF(r,q) $\equiv$ initial fill rate assuming every deficit is zero, the reorder point is r, and the reorder quantity is q.

$$G(r,q) \equiv IF(r,q) \cdot q$$

$$Z_1 \equiv Ex\,(Z \mid Z < R)$$

2. Approximation

Full understanding of this section requires a review of Silver's (6)

paper. From Silver's paper (equation top of p.96), it follows

$$(1) \qquad G(r,q) = q \sum_{x=o}^{r} f(x) + \sum_{x=r+1}^{r+q} (r+q-x)f(x)$$

By using the function G(r,q), Silver's equation (11) can be revealingly rewritten as:

$$(2) \qquad \text{Initial Fill Rate} = \frac{1}{Q+E(Z)} \sum_{z=o}^{R} t(z)G(R-z,Q+z) + pr(z > R)G(0,R+Q)$$

or

$$\frac{1}{Q+E(Z)} \sum_{z=o}^{R-1} t(z)G(R-z,Q+z) + pr(z \geq R)G(0,R+Q)$$

The first step in the approximation is to set

$$(3) \qquad \sum_{z=o}^{R-1} t(z)G(R-z,Q+z) \approx Pr(z < R)G(R-Z_1,Q+Z_1)$$

The second step in the approximation is to represent the distribution of the number of units demanded per transaction by the geometric distribution. For the geometric it is known (cf Silver, Ho, Deemer) that the distribution of the deficit is the same as the distribution of transaction size plus 1; i.e. if W is the amount by which inventory exceeds R,

$$[h(s+w-1) \mid s > w-1] = h(s) = t(s-1)$$

Defining the geometric function:

$$h(s) \equiv pq^{s-1} \qquad\qquad q+p=1$$

Then*:

$$(4a) \qquad \sum_{s=a}^{b} h(s) = p \sum_{s=a-1}^{b-1} q^{s} = q^{a-1} - q^{b}$$

*I first saw these results in notes by Richard Urbach.

(4b) $$\sum_{s=a}^{b} s\ h(s) = p \sum_{s=a}^{b} s\ q^{s-1} = p \frac{d}{dq} \sum_{s=a}^{b} q^{s}$$

and omitting some algebra (found in Appendix)

$$= q^{a-1}\ [a+q/p] - q^{b}[(b+1) + q/p]$$

Using Equations (2), (3) and (4) and some algebra (found in Appendix)

(5) Initial Fill Rate $\approx$

$$\frac{1}{Q+E(s)-1} \left\{(1-q^{R+1})\ G(R-Z_1, Q+Z_1) + q^{R+1} G(0,R+Q)\right\}$$

$$Z_1 = \frac{1}{1-q^{R}} \left\{\frac{1}{p} - q^{R}\ [R + \frac{1}{p}\ ]\right\}$$

3. <u>Computational Considerations</u>

<u>Programming Equation (5)</u>. The Negative Binomial probability mass function is easily calculated by computer. If the function is defined

(6) $$f(x;w,u,v) = u^{w} v^{x} \binom{x+w-1}{x}$$

$u+v=1.$

$$\text{Mean} = \frac{wv}{u}$$

$$\text{Var} = \text{MEAN}/u$$

Then:

(7) $$f(o;\ w,u,v) = u^{w}$$

$$f(x;\ w,u,v) = f(x-1;\ u,v,w)(v)(x+w-1)/x$$

One problem which does arise is that $R-Z_1$, $Q+Z_1$ need not be integer while the negative Binomial is defined only for integer values. Our solution was to interpolate: e.g. if "fr" is a fractional value

$$(8) \qquad f(x+fr; w,u.v) = (fr)\, f(x+1;w,u,v) + (1-fr)\, f(x;w,u,v)$$

Camp Paulson Transformation. The Negative Binomial Cumulative Distribution Function may be approximated by the Normal, provided the value at which the c.d.f. is evaluated is transformed in the manner devised by Camp Paulson.* (In other words if we wish to know the probability demand is $\leq X$, we transform X to X' and find the probability a standard normal is $\leq X'$). Furthermore[2],

$$(9) \qquad \sum_{x=o}^{B} x\, f(x;w,u,v) = \frac{wv}{u} \sum_{x=o}^{B-1} f(x;w+1,u,v)$$

Letting $B = M$, and then R, we see that $G(r,q)$ can be expressed as a function of Negative Binomial c.d.f.'s, and then evaluated using the Camp Paulson approximation. The Camp Paulson approach is useful when lead time demand can be high, and incidentally eliminates any need to interpolate to treat non-integer values as in equation (8).

Computational Comparison of Approaches. Even if the deficit is ignored $G(r,q)$ must be computed. The approximate algorithm requires evaluation of $G(r,q)$ at two different points and not much extra programming. As compared to the exact approach, it eliminates the need to compute the deficit distribution, and to evaluate $G(r,q)$ at a possibly large number of different points.

4. Evaluation

The evaluation compared:

"Exact"	Silver's formula
"Approx"	Approximation as given in equation (5)

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* See reference (1) or (2). In (1), the minus sign before 2nd term in equation (7) definition of z, should read plus.

"Approx/CP" Approximation using Camp Paulson

"Ignore" Use IF(R,Q), ignoring the impact of the deficit.

"Ignore" might also have been characterized as "Poisson-Extended", as an alternative derivation of "Ignore" is:

a. Write down the expression for initial fill under Poisson demand.

b. Replace the Poisson c.d.f. with the Compound Poisson c.d.f. (or perhaps some other such as Normal or Laplace).

The equivalence of the result from step (b) and equation (1) can be shown with some algebra and calculus (see Appendix). As "Poisson Extended", "Ignore" is commonly incorporated into inventory models; e.g., see (2), (5).

Inspection of the tabulated results of the evaluation confirms that when $\overline{S}$ is large relative to lead time demand, the deficit distribution must be considered. Silver calls such items "erratic demand" items. "Approx" and "Approx/CP" work equally well. However, their error does not decrease as the deficit distribution becomes less important - focus on the case where $\overline{X} = 10$, $\overline{S} = 2$ - even though in the limit as $S \rightarrow 1$, "Approx" and "Ignore" must converge to "Exact".

The impact of equation (3), taken by itself, is to produce results between "Exact" and "Ignore". When more of the deficit distribution is larger than R, equation (3) has less impact, so with the exception of the (10,2) cases, approx does better for $R = \overline{X}$ than for $R = 2\overline{X}$.*

The other aspect of the approximation is use of the Geometric to characterize transaction sizes instead of the Logarithmic distribution implied by use of the Negative Binomial (4) and incorporated into "Exact". Additional runs on the (10,2) cases showed this was not a problem; in fact, the impact of the Geometric by itself was to significantly understate initial fill.

Still other runs not shown verified that Approx worked well for expected demand and order size of (25,2), (25,5), (25,25), (50,2), (50,5), (50,25).

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*Even for the (10,2) case, if error is measured as a % of unfilled demand (e.g. .9%/(100%-94.1%), Approx does better when $R = \overline{X}$.

COMPARISON OF APPROACHES

$\bar{X}$	$\bar{S}$	R	Q	"Exact"	"Approx"	"Approx/CP"	"Ignore"
2	2	$\bar{X}$	$2\bar{S}$	70.2%	+0.6%	-0.6%	+11.8%
		$\bar{X}$	$5\bar{S}$	84.2%	+0.4%	-0.3%	+ 6.6%
		$2\bar{X}$	$2\bar{S}$	83.9%	+1.5%	+1.8%	+ 7.4%
		$2\bar{X}$	$5\bar{S}$	91.5%	+0.8%	+0.9%	+ 4.1%
2	4	$\bar{X}$	$2\bar{S}$	64.1%	-0.4%	+0.0%	+24.2%
		$\bar{X}$	$5\bar{S}$	79.8%	+0.3%	+0.3%	+14.0%
		$2\bar{X}$	$2\bar{S}$	72.5%	+0.6%	+1.2%	+19.4%
		$2\bar{X}$	$5\bar{S}$	84.5%	+0.7%	+0.8%	+11.1%
10	2	$\bar{X}$	$2\bar{S}$	62.2%	+2.1%	+2.3%	+ 6.1%
		$\bar{X}$	$5\bar{S}$	75.5%	+1.0%	+1.1%	+ 4.1%
		$2\bar{X}$	$2\bar{S}$	94.1%	+0.9%	+0.8%	+ 1.7%
		$2\bar{X}$	$5\bar{S}$	96.5%	+0.4%	+0.4%	+ 1.0%
10	5	$\bar{X}$	$2\bar{S}$	60.1%	+1.2%	+1.9%	+16.3%
		$\bar{X}$	$5\bar{S}$	74.4%	+1.1%	+1.3%	+10.9%
		$2\bar{X}$	$2\bar{S}$	79.6%	+3.0%	+3.1%	+ 9.9%
		$2\bar{X}$	$5\bar{S}$	87.0%	+1.9%	+1.8%	+ 6.4%
10	10	$\bar{X}$	$2\bar{S}$	56.8%	-0.7%	+0.1%	+26.7%
		$\bar{X}$	$5\bar{S}$	72.5%	+0.4%	+0.7%	+17.4%
		$2\bar{X}$	$2\bar{S}$	69.4%	+1.0%	+0.6%	+20.2%
		$2\bar{X}$	$5\bar{S}$	80.3%	+1.4%	+1.6%	+13.2%

5. Concluding Remarks

The approximation described in this note is not fully satisfying because it depends in part on the ability of the geometric to characterize transaction size. At the same time there are many items for which it is useful, and for which ignoring the deficit distribution is unthinkable.

In some applications the variability of lead time demand is increased as an approximate way of coping with lead time variability or uncertainty about the parameters of the lead time demand distribution. This is readily accommodated by the approximation, since it need only change the calculation of $G(r,q)$.

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APPENDIX

Backup to Equation 4b

$$p\frac{d}{dq}\sum_{x=a}^{b} q^{x} = p\frac{d}{dq}\frac{1}{1-q}\left[q^{a} - q^{b+1}\right]$$

$$= p\frac{\left[a\,q^{a-1} - (b+1)\,q^{b}\right](1-q) + q^{a} - q^{b+1}}{(1-q)^{2}} =$$

$$a\,q^{a-1} - (b+1)q^{b} + \frac{q^{a}-q^{b+1}}{p} =$$

$$q^{a-1}\left[a + q/p\right] - q^{b}\left[(b+1) - q/p\right]$$

Backup to Equation (5), Z_1

$$\frac{\sum_{s=1}^{R} s\,h(s)}{\sum_{s=1}^{R} h(s)} = \frac{(1+q/p) - q^{R}\left[R+1 + q/p\right]}{1-q^{R}} =$$

$$\frac{\left(\frac{1}{p}\right) - q^{R}\left[R + \frac{1}{p}\right]}{1-q^{R}}$$

Backup to Equivalence of "Ignore" and "Poisson Extended"

For "Poisson Extended"

$$\text{Availability} = \frac{1}{Q} \sum_{x=R+1}^{R+Q} F(x-1) =$$

$$\frac{1}{Q} [QF(R) + \sum_{x=R+2}^{R+Q} \sum_{y=R+1}^{X-1} f(y)]$$

$$= \frac{1}{Q} [QF(R) + \sum_{x=R+1}^{R+Q-1} \sum_{y=R+1}^{X} f(y)]$$

Reversing the order of summation

$$\frac{1}{Q} [QF(R) + \sum_{y=R+1}^{R+Q-1} \sum_{x=y}^{R+Q-1} f(y)] =$$

$$F(R) + \frac{1}{Q} \sum_{y=R+1}^{R+Q-1} (M-1-y+1) f(y) =$$

$$F(R) + \frac{1}{Q} \sum_{y=R+1}^{R+Q-1} (M-y) f(y) =$$

"Ignore"

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